

## STUDY OF THE DYNAMICS OF A WIND TURBINE'S DRIVE TRAIN WITH TEETH DEFECTS

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### Abstract

This work presents the application of a developed multi-body model of a wind turbine with complex drive train. The model is adapted for studying the drive train dynamics in the presence of the most common defects of its elements – a tooth cracking and tooth pitting. These defects are modelled by theoretically justified reduction of the amplitude of the respective stiffness of the planetary gear mechanism. As a result of the studies of the gear and available defects, time series of torsional vibrations of the wind drive train hull are obtained and graphically presented. The vibration signal from the wind drive train hull can be measured and detected in its development relatively easily. Therefore, an immediate opportunity for conducting of justified vibration analysis of the drive train of wind turbines appears. The obtained information is very useful for monitoring and vibration analysis of the state of the elements of the wind turbine gear and it can be used to detect defects at an early (and as it turns out the most important) stage of their development.

**Key words:** drive train, wind turbine, dynamic analysis.

**JEL:** L64.

### Introduction

Minimization of the maintenance costs and unplanned repairs of modern wind turbines is a fundamental requirement, which directly affects their effectiveness. A number of studies [10, 20] suggest that one of the most vulnerable and difficult for maintaining and repairing elements of the wind turbines is its drive train. It includes a complex system of gears. Their technical state and working capacity determine the reliability and performance of the drive train and it affects the whole wind turbine.

The main failures, related to wear and damage of the gears, are defined in the International Document ISO 10825 "Gears. Wear and damage to gear teeth. Terminology". It is adopted in most European countries. An identical edition in Bulgarian language is approved by the Bulgarian Institute for Standardization as ISO Standard 10825. It is applicable for testing and fault diagnosis of gears.

This International Standard is a terminology issue which creates unity in the establishment of the state of the gears and the type of damages after a certain period of operation. But this standard does not describe the possible reasons for these damages and it does not offer preventive measures in order to avoid damages. Moreover, there are no specific values for identification of particular conditions and there are no defined moments for the need of repair works. Therefore, it is necessary to conduct additional advanced studies in specific applications of gears. These studies are based on the experience gained in their practical use.

A large number of types of tooth damages of the gear are classified in the cited standard. The most frequent damage of the teeth is a tooth cracking

and formation of holes on their surface (tooth pitting). Exactly these two teeth's damages of the gears in wind turbine drive train are the subject of the study in this research.

Breaking a tooth is considered as the most dangerous damage that can lead to very serious consequences for the mechanism. A photo of a broken tooth is shown in Fig. 1. Fundamentally broken teeth are possible with: impact (single) load; high overload; plastic deformation (cold outflow); external (foreign) bodies got into gear engagement. The last stated factor leads to the formation of bending stresses at the base of the tooth that are greater than the strength of the material. A granular fracture of the destruction surface characterizes all cases examined above.



**Fig. 1. Photo of tooth cracking**

The formation of the holes (pitting) of the teeth is a fault, in which the metal particles break off from the tooth surface and form the scattered holes with dark bottom (without metallic glitter) across the en-

tire width of the tooth (usually in the pole line). A photo of teeth with pitting is shown in Fig. 2. Pitting may be initial, progressive and micropitting. The pitting is a result of the surface fatigue associated with sufficiently large cyclic contact stresses. It can be seen in the gears working with plenty lubrication.



Fig. 2. Photo of tooth pitting

The modern wind turbines have a complex planetary gearbox. Studies on the vibrations in a planetary gear system have been carried out in [1, 3, 7, 8]. The tooth meshes are modelled as a linear spring with stiffness that is a time function. For this reason the vibration equations of a planetary gear system are differential equations with periodic coefficients, [7, 8, 12].

The applications of these modelling techniques on different drive trains of wind turbines are presented in [1, 4, 5, 6, 9, 11]. References [13, 14] present the numerical investigations of the given wind turbine in this paper, where the meshes stiffness are modelled as constant springs. In this case the differential equations, which describe the torsional vibrations of the wind turbine, have constant coefficients. In [15, 16, 18], a dynamic model of wind turbine is proposed, where the mesh stiffness is modelled as a time function, and the aerodynamic and electromagnetic torques are constant. In Ref. [17] the same dynamical model is proposed, but the aerodynamic and electromagnetic torques are modelled as time functions.

References [2, 3, 14, 19, 21, 22] present the effects of tooth damages and the wearing on the gear dynamics. Typical gear failures are: broken tooth, cracked tooth (Fig. 1), worn tooth, pitting (Fig. 2), spalling and chipping. In these references several approaches are proposed for understanding the influence of the local damages on the dynamic behaviour of the gearbox.

### 1. Dynamic model of wind turbine

The wind turbine consists of a rotor, a drive train and a generator (Fig. 3).

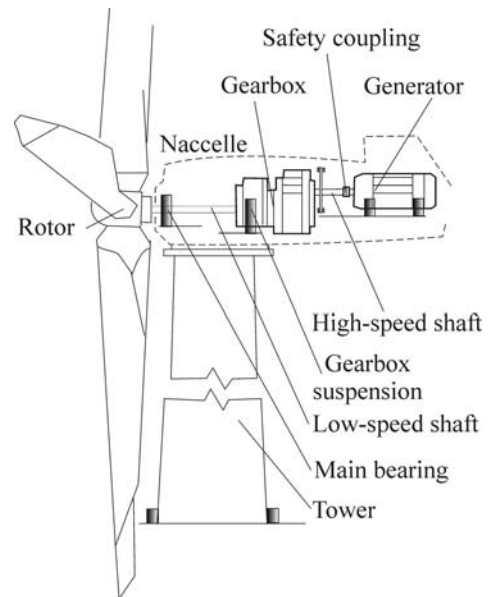


Fig. 3. Schematic sketch of wind turbine

The drive train has a gearbox with three stages. The gear stages include two high-speed parallel gear stages and a low-speed planetary gear stage (three identical planets with spur teeth, sun and fixed ring wheel) (Fig. 4).

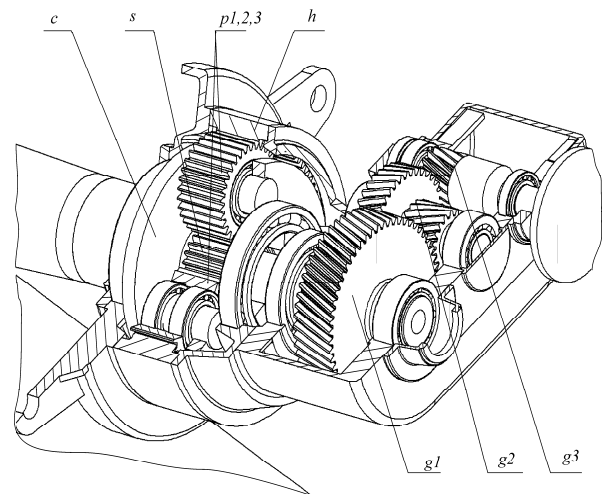


Fig. 4. Sketch of gearbox

Legend: *h* – hull, *c* – carrier, *p*<sub>1,2,3</sub> – planets, *s* – sun, *g*<sub>1,2,3</sub> – gears

The dynamic multi-body model is shown in Fig. 5. It consists of a rotor with 3 rigid blades, a low-speed elastic shaft, a gearbox with 3 gear stages, a high-speed elastic shaft and a generator rotor. Thus, the model consists of 10 bodies and 11 DOF's.

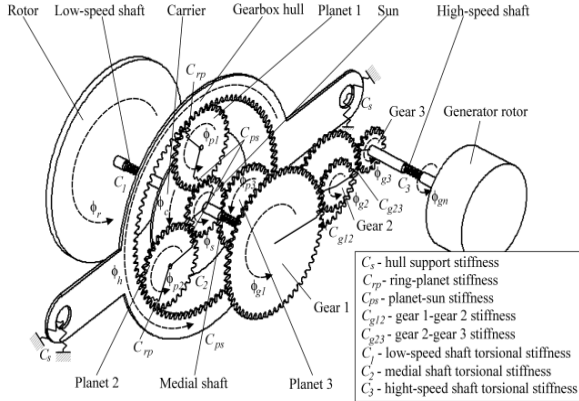


Fig. 5. Dynamical model of wind turbine

The differential equations, describing the torsional vibrations of the wind turbine, are

$$M\ddot{q} + [C - \omega^2 C_\omega]q = T, \quad (1)$$

where

$q$  is the degrees of freedom vector

$$q = [\phi_h \ \phi_c \ \phi_r \ \phi_{p1} \ \phi_{p2} \ \phi_{p3} \ \phi_s \ \phi_{g1} \ \phi_{g2} \ \phi_{g3} \ \phi_{gn}]^T \quad (2)$$

$M$  – the inertia matrix,

$C$  – the stiffness matrix,

$C_\omega$  – the centripetal stiffness matrix.

The angles  $\phi_i$  ( $i=h, c, r, p1, p2, p3, s, g1, g2, g3, gn$ ) are the rotational angles of the ring (gearbox hull), carrier, rotor (hub), planet 1, planet 2, planet 3, sun, gear 1, gear 2, gear 3 and the generator rotor (Fig. 5).

The vector of the external forces  $T$ , caused by the wind and the electricity grid, is

$$T = [0 \ 0 \ T_{aero} \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ T_{gen}]^T \quad (3)$$

The non-zero numbers of inertia matrix  $M$ , stiffness matrix  $C$ , and  $C_\omega$  can be seen in [15, 16].

## 2. Modelling of the Gear Mesh Stiffness

### 2.1. Modelling of the healthy (perfect) gears

The gear contact forces between wheels are modelled by linear spring acting in the plane of action along the contact line (normal to the tooth surface) [3, 7, 8]. The stiffness gear is defined as a normal distributed tooth force in a normal plane causing the deformation of one or more engaging tooth pairs, over a distance of  $1 \mu\text{m}$ , normal to an involute profile in a normal plane, [DIN 1987]. This deformation is a result from the bending of the teeth in contact between the two gear wheels, the first of which is fixed and the other is loaded. The stiffness varies in the time length and can be expressed in a time Fourier series form, [7]. Each mesh stiffness is presented by

$$C_{g_i}(t) = C_{g_i} + C_{g_{iv}}(t) \quad (4)$$

where

$C_{g_i}$  and  $C_{g_{iv}}$  are mean and time-varying components of the stiffness.

The variation part is periodic with frequency  $\Omega_i = Z_i \omega_i$  ( $Z_i$  is the number of teeth on the gears,  $\omega_i$  is mean angular velocity of the gear shafts) and it is expressed in Fourier series as

$$C_{g_{iv}} = 2C_{g_{vs}} \sum_{s=1}^{\infty} (a_s \sin s \Omega_i t + b_s \cos s \Omega_i t), \quad (5)$$

where

$$a_s = -\frac{2}{s\pi} \sin[s\pi(\varepsilon - 2p)] \sin(s\pi\varepsilon)$$

$$b_s = -\frac{2}{s\pi} \cos[s\pi(\varepsilon - 2p)] \sin(s\pi\varepsilon)$$

Without loss of generality, it can be accepted that  $p=0$  ( $p$  is the phasing between planets) [7]. In practice, three or four Fourier terms reasonably approximate the stiffness variation.

The rectangular waves are often used to approximate the mesh stiffness between 2 pairs of teeth in contact [3, 8]. In this study, the  $C_{g_{iv}}$  are specified as rectangular waves with variational amplitudes and periods.

The rotating frequency of the carrier, sun, planets, and mesh frequency for the planetary gear stage can be calculated by

$$\Omega_c = \omega_c / 2\pi, \quad (6)$$

$$\Omega_s = \omega_c (Z_s + Z_r) / Z_s, \quad (7)$$

$$\Omega_p = (\omega_s Z_s - \omega_c (Z_s + Z_p)) / Z_p, \quad (8)$$

$$\Omega_{\text{mesh}} = \Omega_c Z_r. \quad (9)$$

### 2.2. Modelling of the gears with tooth defects

It has been established [3] that gear tooth failure will induce amplitude and phase changes in vibration, which in turn can be represented by magnitude and phase changes in gear mesh stiffness.

The tooth-crack-induced variations of mesh stiffness, used for this simulation, are shown in Fig. 6.

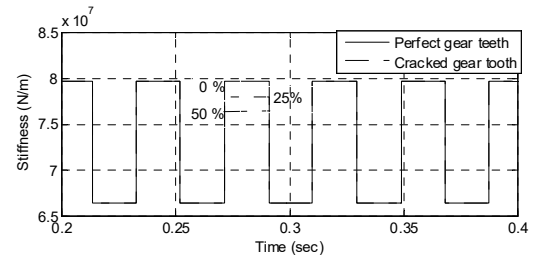


Fig. 6. Tooth cracking effect on mesh stiffness

The tooth-pitting-induced variations of mesh stiffness, used for this simulation, are shown in Fig. 7.

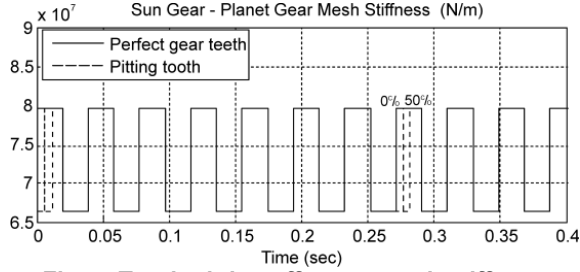


Fig. 7. Tooth pitting effect on mesh stiffness

These defects-induced changes lead to an amplitude modulation of the gear mesh signal. The new mesh signal, resulting from defect modelling, can be expressed by

$$C_{g_d}(t) = C_{g_i}(t)d(t), \quad (10)$$

where

$d(t)$  is the modulating function.

As a result of this amplitude modulation, an exciting force appears, and its frequency affects the system [3].

Assuming localized tooth crack and tooth pitting on only one sun tooth, the defect frequency is written as

$$\Omega_d = 3(\Omega_s - \Omega_c). \quad (11)$$

### 3. Results

All calculations are accomplished by using the codes of MATLAB.

The drive train data can be seen in [14, 15]. It is assumed that the aerodynamic torque and electromagnetic torque are  $T_{aero} = -T_{gen} = 15000$  Nm. The rotor is turned with angular velocity  $\omega = 18$  tr/min.

It is assumed that there is a cracked tooth of the sun of the planetary gear stage. The rates of degradation of stiffness are respectively 25% and 50%. Fig. 8 presents the time series of torsional vibrations of the wind drive train hull. This figure illustrates the influence of cracked tooth of the sun of the planetary gear stage on the vibration signals for the both degradation values. It is seen that the greater degradation of the stiffness leads to the obvious periodical impulses caused by the cracked tooth.

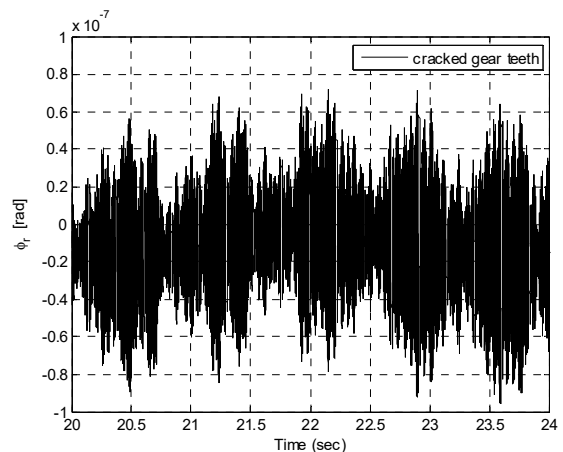
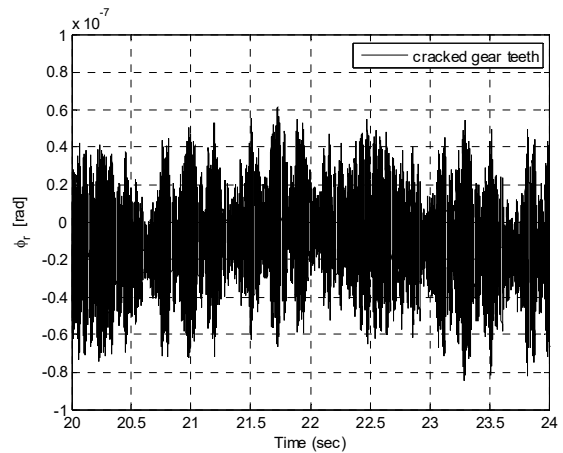
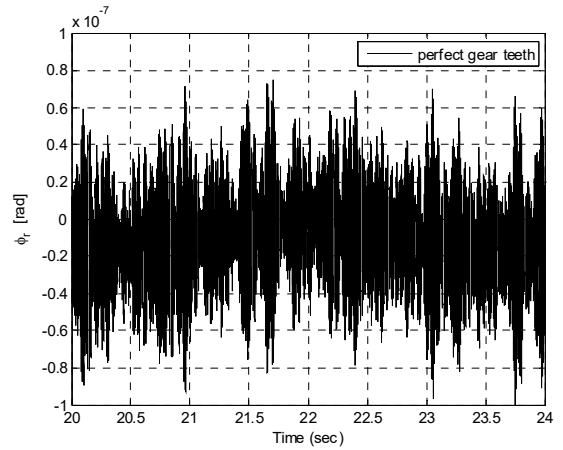
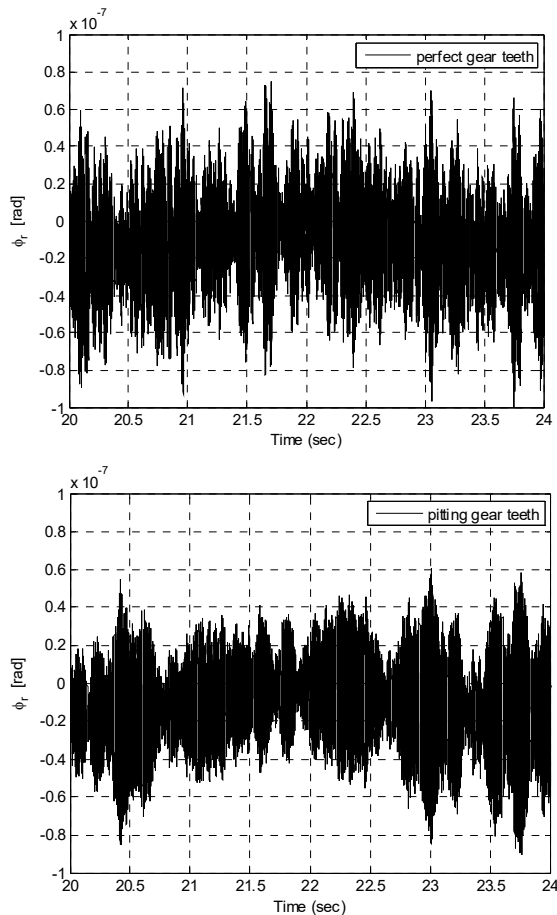


Fig. 8. Hull torsional vibration – perfect gear, sun cracked tooth with degradation of stiffness 25% and 50%

It is assumed that there is a pitting tooth of the sun of the planetary gear stage. The rate of degradation of stiffness is 25%. Fig. 9 presents the time series of torsional vibrations of the wind drive train hull. It becomes clear that the obvious periodical impulses, caused by the pitting tooth, appear.

All obtained results are useful for detailed analysis of the vibration behaviour of mechanical transmission of wind turbines in the presence of the

relevant defects. These results can be used as a major and reliable source of information during the conducting vibration analysis of the wind turbines mechanical gear.



**Fig. 9. Hull torsional vibration – perfect gear and sun pitting tooth**

### Conclusion

The obtained results confirm the applicability of the developed multi-body model of wind turbine for research on the dynamics of its complex drive train. The adapting of the model for studying the drive train dynamics in the presence of the most common defects of its elements – tooth cracking and tooth pitting – requires theoretically justified reduction of the amplitude of the respective stiffness of the planetary gear mechanism.

Time series of torsional vibrations of the wind drive train hull are obtained and graphically presented as a result of the studies of the gear in the presence of defects. The vibration signal from the wind drive train hull can be measured and detected in its development relatively easily. Thus, an immediate opportunity appears to conduct justified vibration analysis of the drive train of wind turbines. Therefore, it becomes possible to detect defects at an early stage of their development and even in

their generation. In this way the efficiency of the wind turbine is guaranteed.

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## ИЗСЛЕДВАНЕ НА ДИНАМИКАТА НА МУЛТИПЛИКАТОР НА ВЕТРОГЕНЕРАТОР С ДЕФЕКТИ НА ЗЪБИТЕ

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### Резюме

В тази работа е представено приложението на разработен многомасов модел на ветрогенератор със сложна механична предавка. Моделът е адаптиран за изследване на динамиката на предавката при наличие на най-често срещаните дефекти в елементите ѝ – напукан зъб и наличие на питинг. Тези дефекти се моделират с теоретично обосновано намаляване на амплитудата на коравината на съответното зъбно колело от планетарния механизъм. В резултат на проведените изследвания на работата на предавката при наличие на дефекти са получени и нагледно представени виброграмите на ъглите на завъртане на корпуса на предавката. Вибрационният сигнал от корпуса на мултипликатора сравнително лесно може да бъде измерен и проследен в неговото развитие. Така се оказва, че се открива непосредствена възможност за провеждане на обоснована вибродиагностика на механичната предавка на ветрогенератора. Получената информация е много полезна за системите за мониторинг и вибродиагностика на състоянието на елементите на предавката на ветрогенератора и може да се използва за откриване на дефекти в ранен (и както се оказва най-важен) етап от тяхното развитие.

**Ключови думи:** мултипликатор, ветрогенератор, динамичен анализ.